

Lecture 9

Combining Signals & Systems together using IMU as an example

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7 things you have learned about systems (1)

1. A system can be characterised in terms of differential equations in the time-domain, or in terms of transfer functions in the Laplace domain.
2. Laplace transform converts differential equations into algebraic equations. While in Fourier, we use the frequency variable $j\omega$, in Laplace, we use the complex frequency variable s , where $s = \alpha + j\omega$.
3. Transfer function $H(s)$ is the relationship between output $Y(s)$ and input $X(s)$ in the Laplace domain:

$$Y(s) = H(s) \times X(s)$$

4. Frequency response $H(j\omega) =$ Transfer function $H(s)$ evaluated at $s = j\omega$:

$$H(j\omega) = H(s) \Big|_{s=j\omega}$$

7 things you have learned about systems (2)

5. Fourier transform and frequency response are only valid of **steady-state conditions**; Laplace transform and transfer function are useful for **both steady state and transient conditions**.
6. Non-linear systems can be approximated as linear if you operate over a small signal range.
7. A general 2nd order system can be expressed in term of damping factor ζ and natural frequency ω_0 , and it can be under-damped, critically-damped or over-damped.

Topics omitted:

- ◆ Ideas of poles and zeroes of a system
- ◆ How poles and zeroes affect steady-state and transient behaviour of a system
- ◆ Stability issues of a system

1 & 2 - Differential Equation vs Transfer Function

- ◆ We use differential equations to model systems in time-domain:

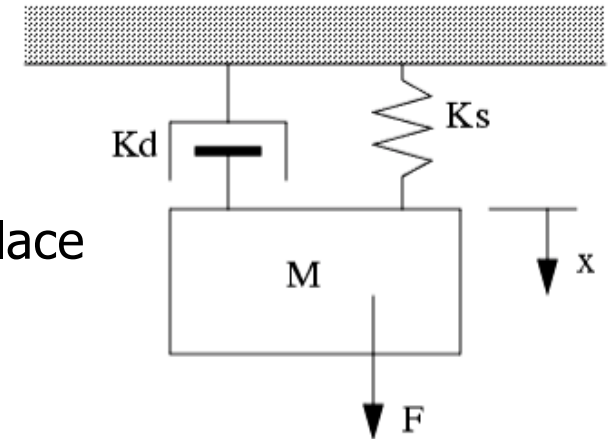
$$M \ddot{x}(t) + K_d \dot{x}(t) + K_s x(t) = F(t)$$

- ◆ We use transfer functions to model systems in Laplace domain as algebraic equation:

$$Ms^2 X(s) + K_d s X(s) + K_s X(s) = F(s)$$

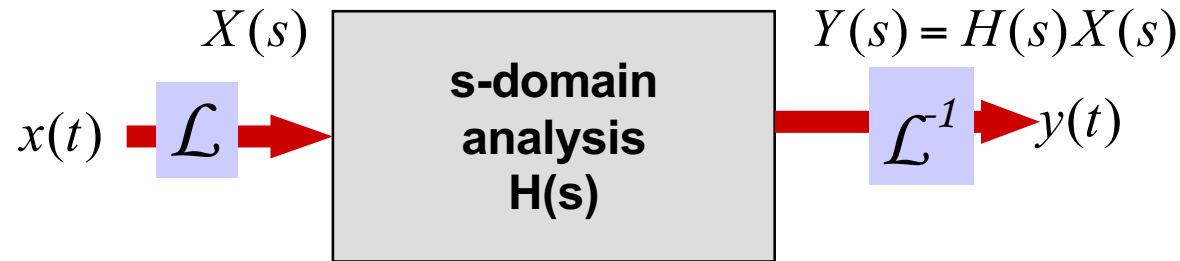
$$(Ms^2 + K_d s + K_s)X(s) = F(s)$$

$$\Rightarrow H(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + K_d s + K_s}$$



$$s^k \longleftrightarrow \frac{d^k}{dt^k}$$

3 - Transfer Function $H(s) = Y(s) / X(s)$



- ◆ Once transformed to the s-domain, analysis and prediction of the system become easy if we know the system's characteristic $H(s)$, which is also called the **transfer function**.
- ◆ Transfer function $H(s) = \text{Output } Y(s) / \text{Input } X(s)$

or

$$\text{Output } Y(s) = \text{Transfer Function } H(s) \times \text{Input } X(s)$$

4 – Frequency Response vs Transfer Function

Laplace Transform

$$\mathcal{L}\{x(t)\} = X(s) = \int_0^{\infty} x(t)e^{-st} dt$$

$$s = j\omega$$



Fourier Transform

$$\mathcal{F}\{x(t)\} = X(j\omega) = \int_0^{\infty} x(t)e^{-j\omega t} dt$$

Transfer Function

$$H(s)$$

$$s = j\omega$$



Frequency Response

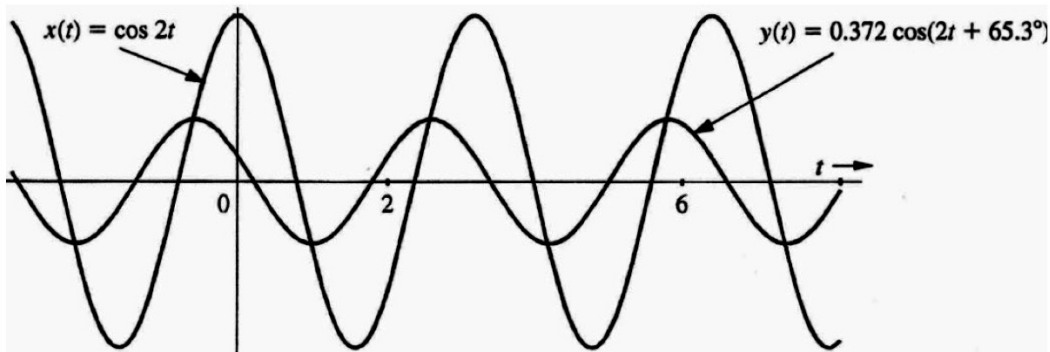
$$H(j\omega)$$

- ◆ We can find the frequency response of a system by substituting $s = j\omega$ into the transfer function:

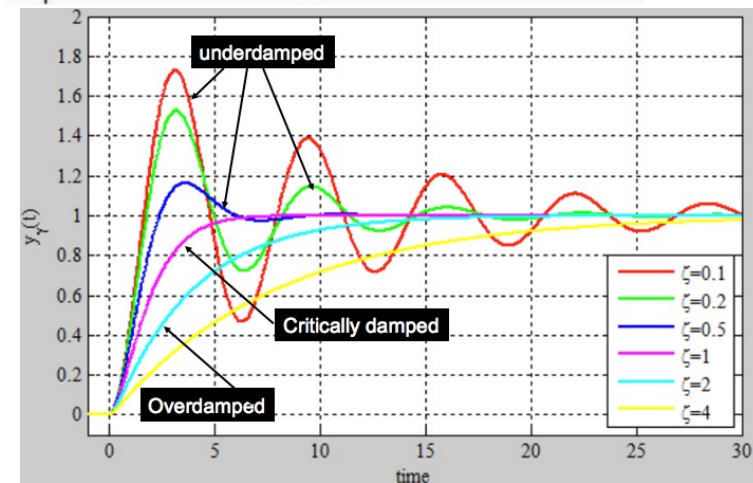
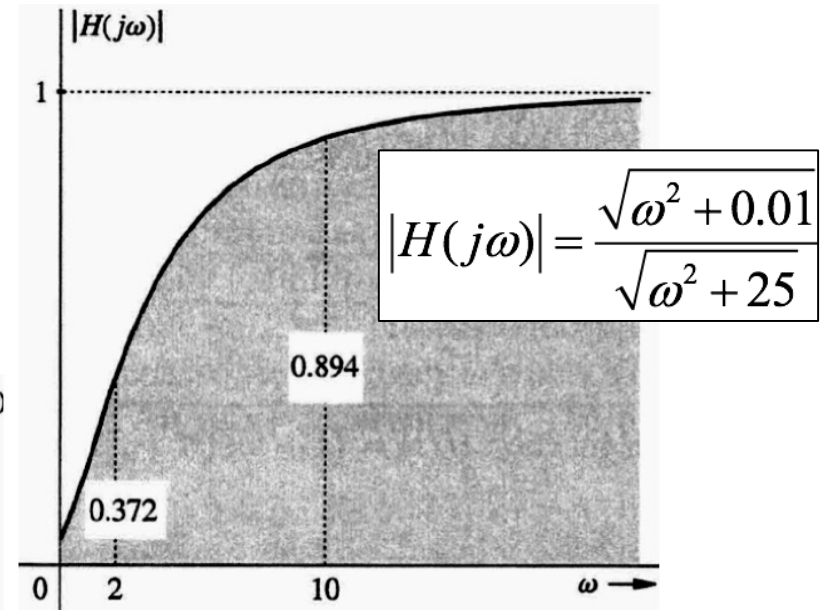
$$H(s) \Big|_{s=j\omega} = H(j\omega) = |H(j\omega)| e^{j \angle H(j\omega)}$$

5 – Steady state vs Transient

- ◆ When using frequency response $H(j\omega)$, we assume inputs are everlasting sinusoids at frequency $j\omega$. That is, we assume that all transient conditions have died down.

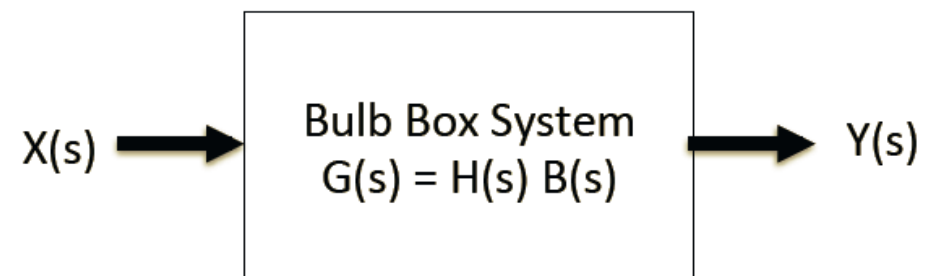
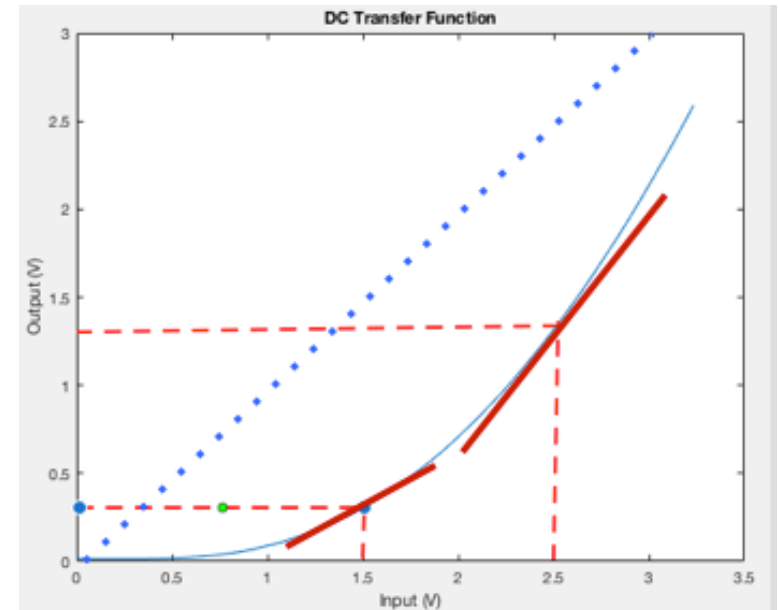


- ◆ When using Laplace transform to model system behaviour, we can model both steady state and transient behaviours.



6 – Linear approximation in small signals

- ◆ A non-linear system such as the Bulb Board can be approximated as linear provided that we only **operate** in small region.
- ◆ We called this the “**operating point**” of the system.
- ◆ Shown here are two operating points, one at 2.5V input, and another at 1.5V input.
- ◆ If we now only use small signal amplitude (sinewaves), then the behaviour of the non-linear system is approximated to be linear.
- ◆ Now, we can use transfer functions to model its behaviour as shown here.



$$G(s) = \frac{1000}{0.038s^3 + 1.19s^2 + 43s + 1000}$$

7 – Damping factor and Natural frequency

- ◆ Let us take the transfer function $H(s)$ of the 2nd order system used in Bulb Board is an example:



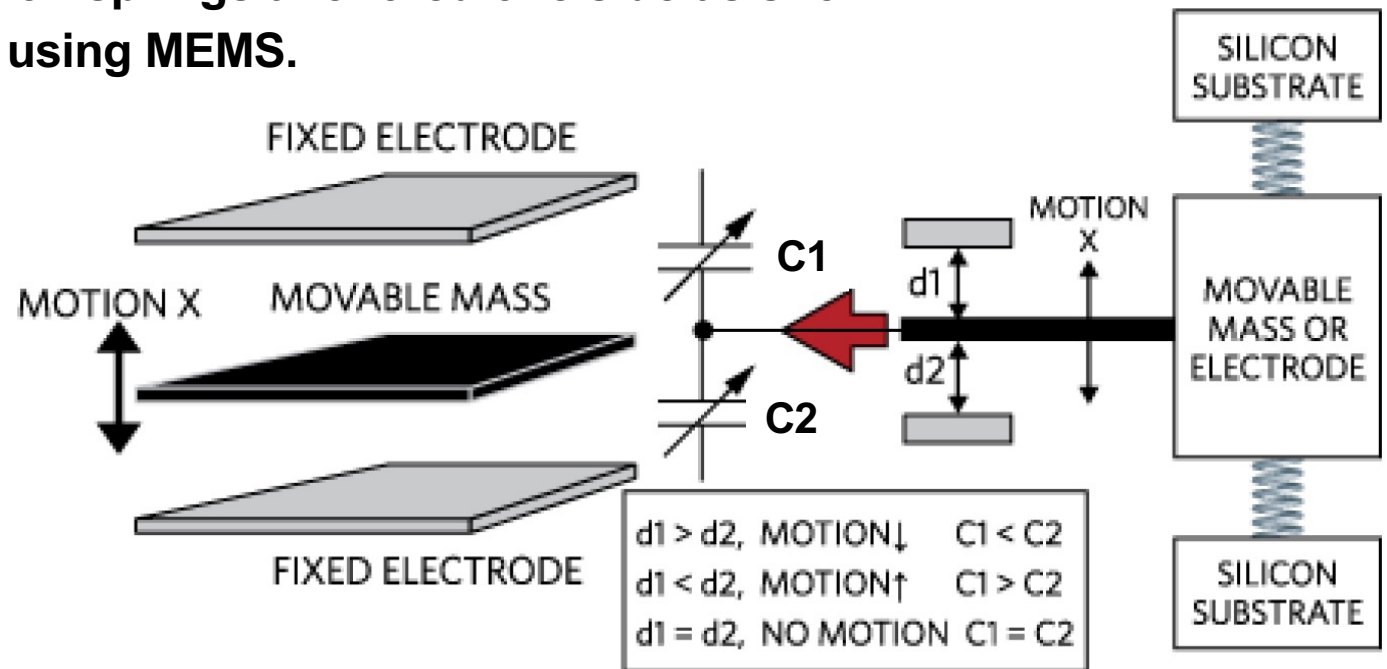
- $\omega_0 = \sqrt{a_0} = 31.62$, **resonant frequency** in rad/sec, or $31.62/2\pi = 5\text{Hz}$
 - $\zeta = \frac{a_1}{2\sqrt{a_0}} = \frac{5}{2\sqrt{1000}} = 0.079$, the **damping factor** (very small, ideal = 1)
 - $K = \frac{b_0}{a_0} = 1$, DC gain of the system at zero frequency
- ◆ Since the damping factor is very small (much smaller than 1), this system is highly oscillatory.

$$H(s) = \frac{b_0}{s^2 + a_1s + a_0} = K \frac{\omega_0^2}{s^2 + 2\zeta\omega_0s + \omega_0^2}$$

Motion Sensing – Accelerometer

Basic Principle

- ◆ Newton's 2nd Law of motion: $F = \text{mass} \times \text{acceleration}$.
- ◆ Sense acceleration is really sensing the force on a mass.
- ◆ Use capacitive sensing with MEMS.
- ◆ Acceleration causes mass to move.
- ◆ Mass pivoted on springs anchored one side as shown.
- ◆ Implemented using MEMS.

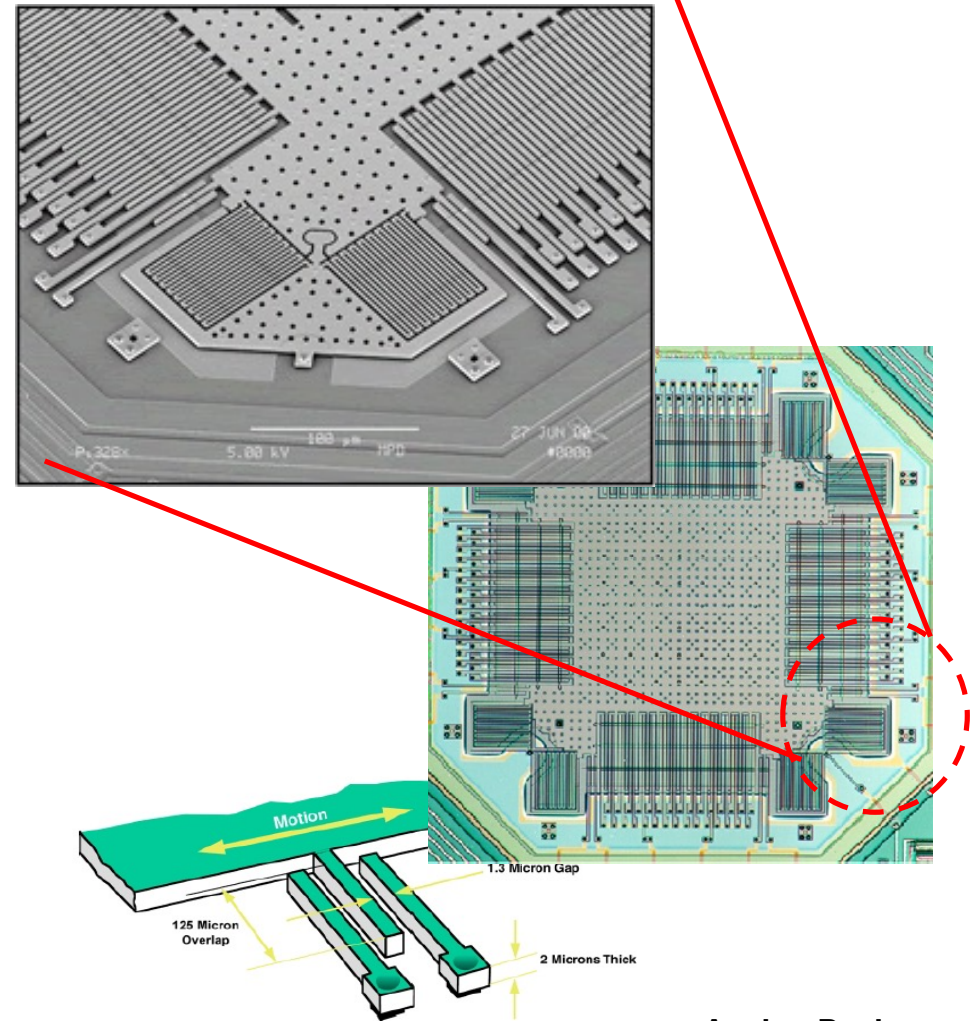
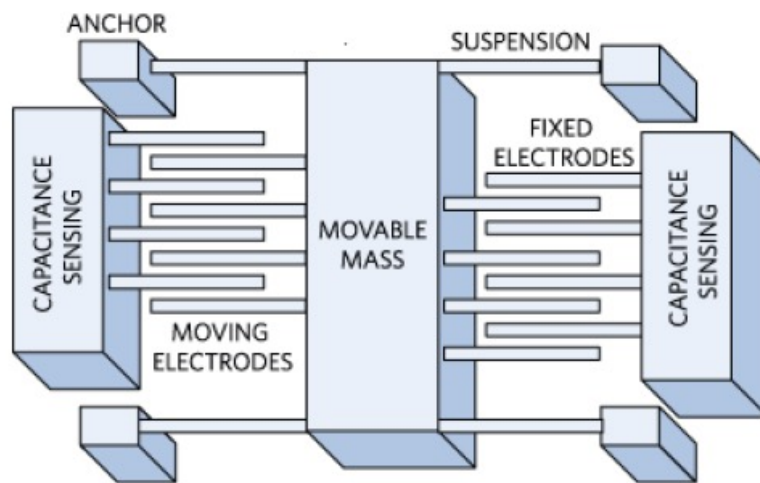


Source: Maxim Integrated

Motion Sensing - MEMS accelerometers

Capacitive MEMS accelerometer

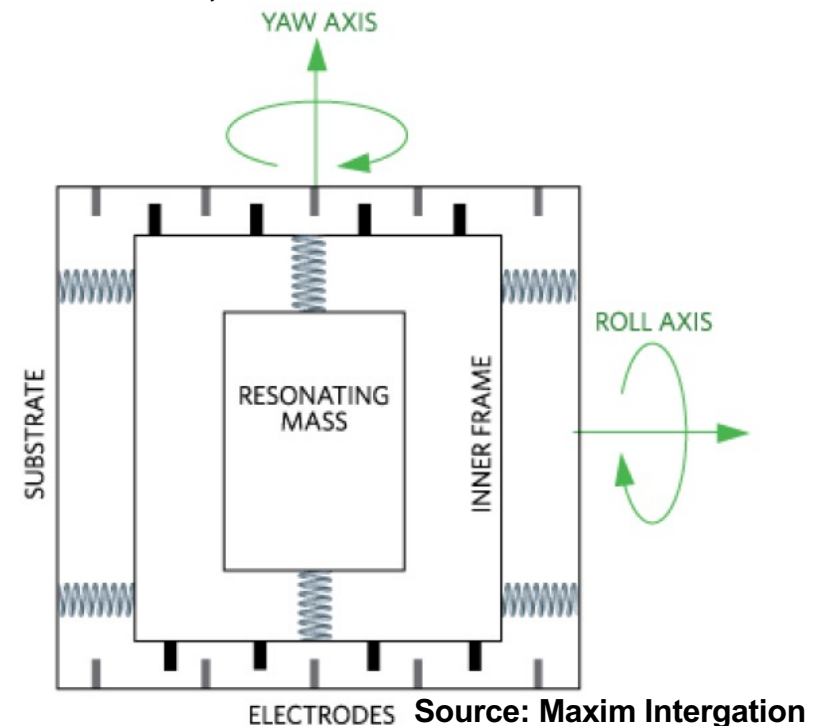
- ◆ The displacement of the movable mass (micrometer) is caused by acceleration.
- ◆ It creates an extremely small change in capacitance for proper detection. Therefore practical sensors use multiple movable and fixed electrodes, all connected in a parallel configuration as shown.



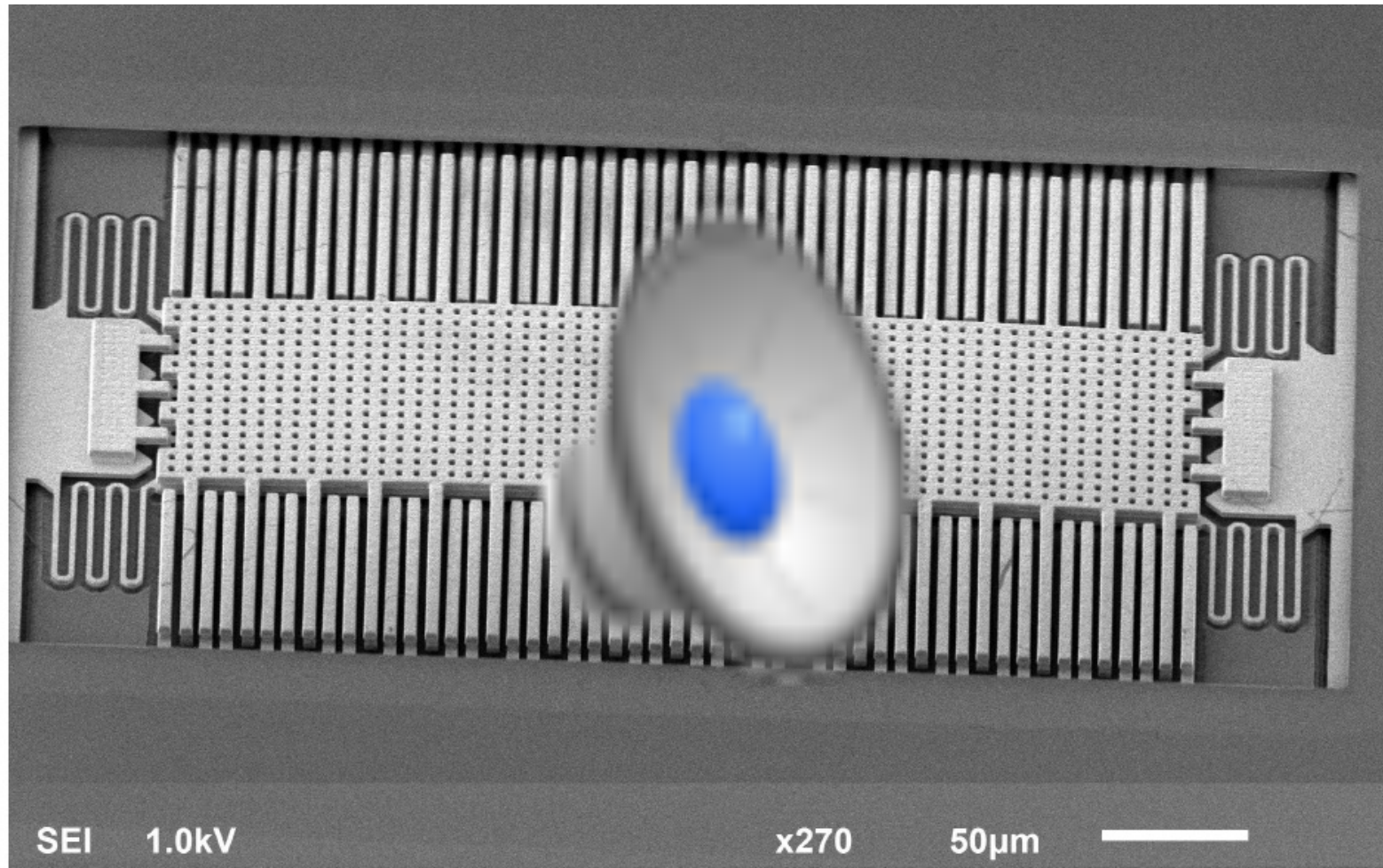
Source: Analog Devices

Orientation Sensing - MEMS gyroscopes

- ◆ Accelerometers measure linear acceleration (specified in mV/g) along one or several axis.
- ◆ A gyroscope measures angular velocity (specified in mV/deg/s).
- ◆ Therefore, the accelerometer's output will not respond to change in angular velocity.
- ◆ However MEMS gyroscopes are similar to accelerometer, but the structure is different as shown here.
- ◆ Here the resonating mass is mounted in an inner frame held by two springs.
- ◆ The inner frame is mounted by springs to the substrate with springs in 90 degrees to the inner springs.
- ◆ Due to the Coriolis Effect, angular rotations in the roll axis and the yaw axis (see diagram) are now translated to linear accelerations.
- ◆ The capacitive fingers are now mounted on the peripherals of the inner frame and the fixed substrate.



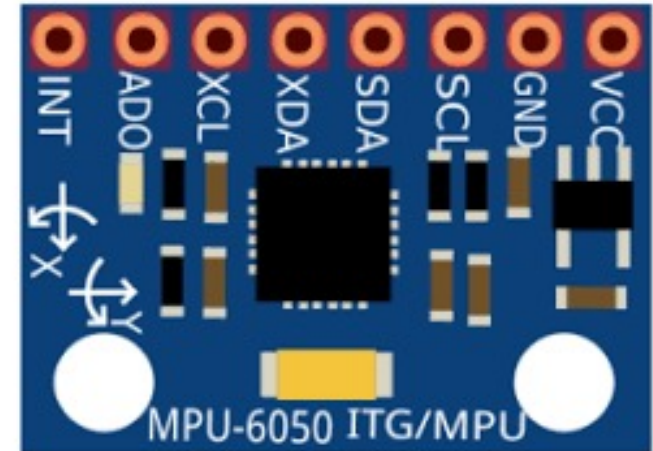
A short video on “MEMS Accelerometer”



A scanning electron microscope photo of a lateral accelerometer
Piotr Michalik et al, IEEE Sensors, Nov 2015

Lab 4 – Task 1: Measuring Angel of tilt – the IMU

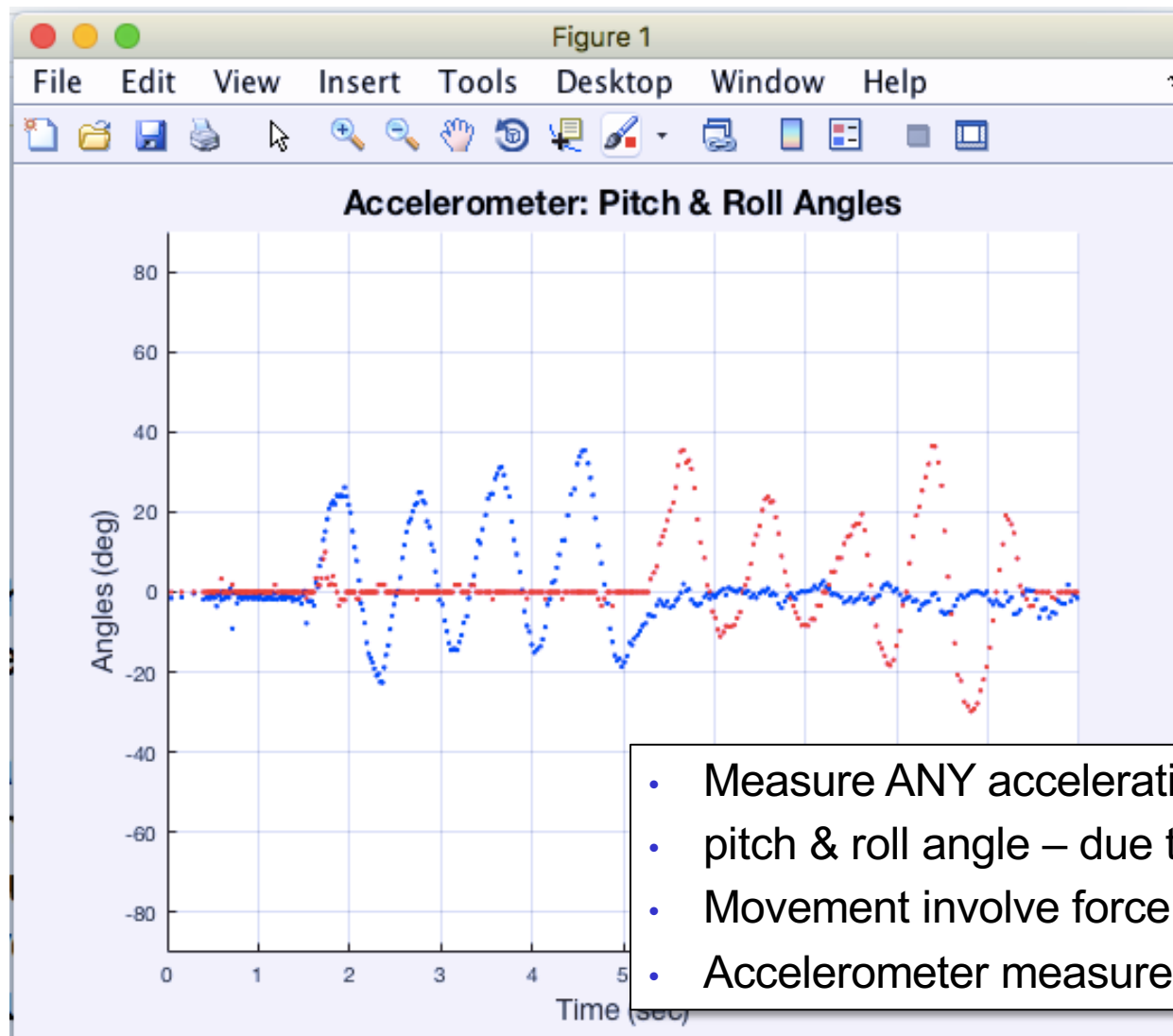
- ◆ The IMU – insertia measurement unit – has built in 3-axis accelerometer and 3-axis gyroscope
- ◆ The module uses I2C interface on two pins: SCL and SDA
- ◆ Easy to access from Matlab using PyBench:



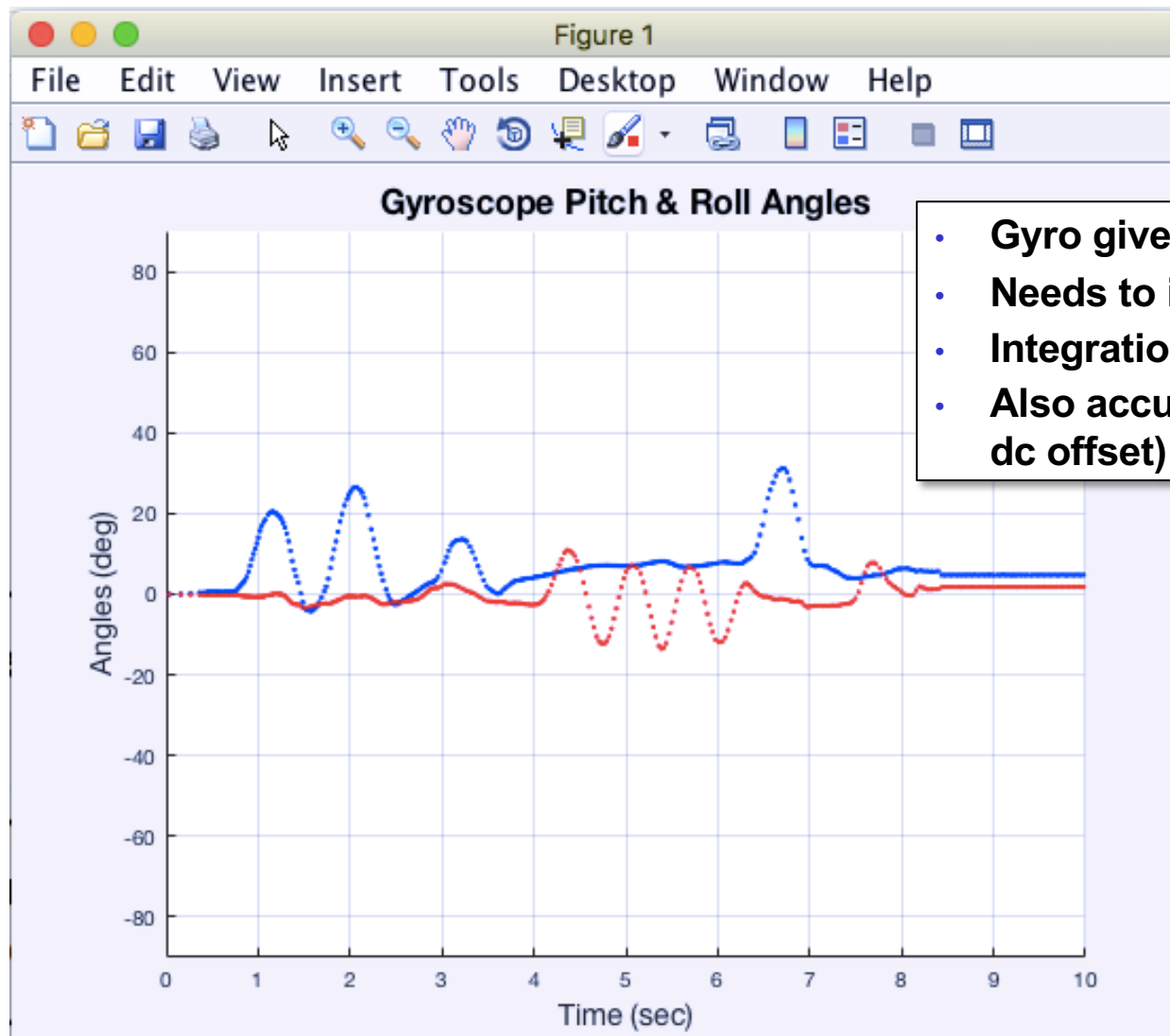
```
[p, r] = pb.get_accel();           % p, r = pitch & roll angle in radians
[x, y, z] = pb.get_gyro();        % x, y, z = rate of rotation in 3-axes in rad/sec
```

- ◆ **Pitch angle p** – plane pointing up or down
- ◆ **Roll angle r** – plane pointing left or right
- ◆ Angle can be in unit radian or degree: $\text{degrees} = \text{radians} * 180 / \pi$
- ◆ Generally use radian for calculations; use degree of display
- ◆ **x, y** an **z** are the angular velocity in the three axes of rotations

Lab 4 – Task 1a: Accelerometer

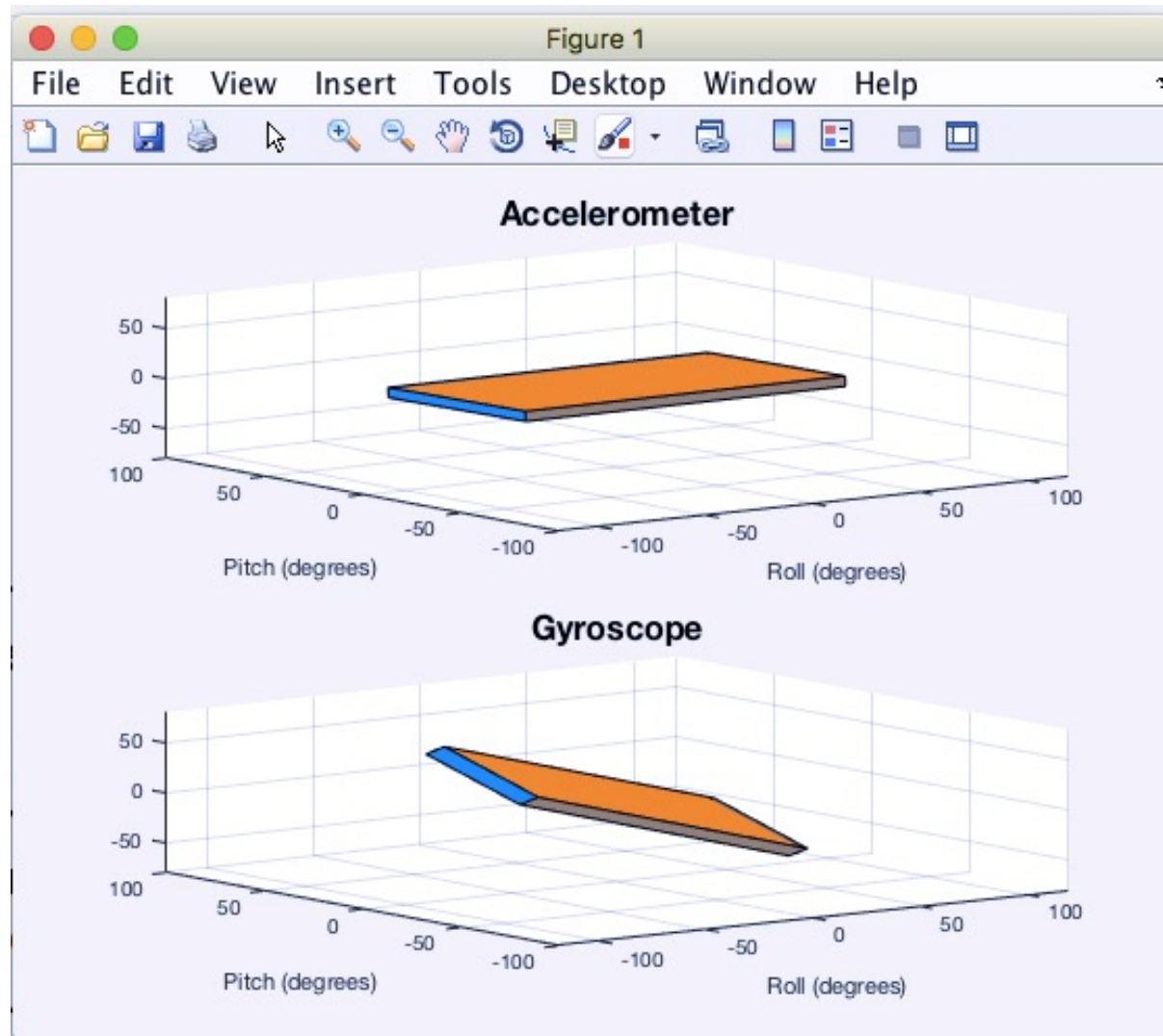


Lab 4 – Task 1b: Gyroscope

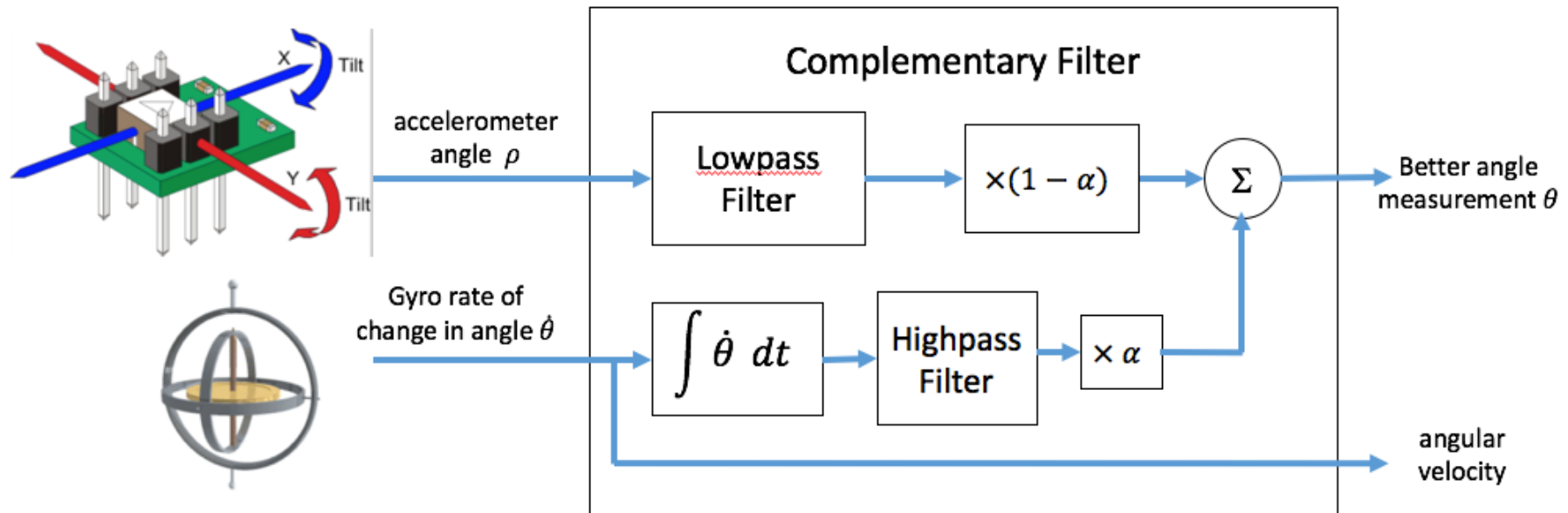


- Gyro gives angular velocity, not angle
- Needs to integrate to get angle
- Integration = accumulation
- Also accumulate errors – causing drift (or dc offset)

Lab 4 – Task 2: 3D visualization



Lab 4 – Task 3: Complementary Filter - Concept



$$\text{angle } \theta_{new} = \alpha \times (\theta_{old} + \dot{\theta} dt) + (1 - \alpha) \times \rho$$

where

α = scaling factor chosen by users and is typically between 0.7 and 0.98

ρ = accelerometer angle

θ_{new} = new output angle

θ_{old} = previous output angle

$\dot{\theta}$ = gyroscope reading of the rate of change in angle

dt = time interval between gyro readings

Three Big Ideas

- ◆ Accelerometer measurement of angle is inherently noisy - it cannot distinguish acceleration due to gravity or due to motion.
- ◆ Gyroscope measurement of angle is inherently "drifty" – gyroscope provides angular velocity measurement. Angle measurement is derived through integration. This results in time varying offset called drift.
- ◆ Much better angle estimation can be obtained by filtering and fusion of the two types of measurements.